MATHEMATICS 241-001 FIRST HOUR EXAM, OCT. 8, 2014
Shatz

There are 5 problems on this exam, answer all of them. The first 3 are multiple choice, and the first of these consists of five true/false statements. **Circle the ENTIRE phrase you deem correct among the choices given and for the true/false statements, circle the ENTIRE word "true" or "false" as the case may be**. There is partial credit given for the answers to the questions numbered 2 through 5–**you must show your work in each part to get any credit for these**. Write answers to ALL questions on the exam paper and be sure to show your computations for the long answer problems. There are no blue books just the exam sheets. Credit scores for each problem are indicated with the problem.

No books, tables, notes, calculators, computers (of any sort), cell-phones or any other electronic gear are allowed. One 3" x 5" index card, HANDWRITTEN (both sides OK) in your own handwriting, is allowed.

Please fill in the data below NOW.

Your name (print please)——————————————.

Your signature——————————————————.

Circle recitaton time: T 9:30 Th 9:30

Your Penn ID number—————————————.

PLEASE DO NOT WRITE BELOW THIS LINE

I

II

III

IV

V TOTAL

MULTIPLE CHOICE QUESTIONS.

I) (20 points-no partial credit) True/False (4 points each answer):

a) If $f(x)$ has a Fourier Series, then that series can be differentiated term by term and the resulting series converges to $f'(x)$.
True False

b) The Laplace Equation $\Delta(u) = 0$ can be thought of as describing the equilibrium temperatures found from the heat equation.
True False.

c) When we solve the heat equation, we always get a function whose terms decay exponentially in time.
True False.

d) The Fourier Series of an odd function on the interval $[-L, L]$ involves just terms of the form $a_k cos(\frac{k\pi x}{L})$.
True False.

e) In solving the heat equation PDE, we need boundary conditions and initial conditions to get a unique answer.
True False.

NB. Here, and in what follows, we use the notation: u_t for $\frac{\partial u}{\partial t}$, u_{xx} for $\frac{\partial^2 u}{\partial x^2}$, etc.

II) (15 points) A certain rod has length 10 and the heat equation for this rod is $u_t = u_{xx} + 1$ (which means heat is produced in the rod). Suppose $u_x(0,t) = 1$ and $u_x(10,t) = \beta$ and we seek the equilibrium temperature distribution, $u(x)$. Then such will exist provided β is:

1) -1

2) -3

3) -5

4) -7

5) -9.

III) (15 points) A rod of length π satisfies the heat equation $u_t = u_{xx}$ with boundary conditions $u(0,t) = u(\pi,t) = 0$ and initial condition $u(x,0) = 50$. The solution of this problem is a function given by the infinite series:

$$u(x,t) = \sum_{k=1}^{\infty} b_k e^{-k^2 t} sin(kx).$$

Compute the coefficient b_1 and use this first term to give an approximation to the temperature of the middle of the rod at time t. Your answer is:

1) $\frac{200}{\pi} e^{-9t}$

2) $\frac{200}{\pi} e^{-t}$

3) $\frac{50}{\pi} e^{-9t}$

4) $\frac{50}{\pi} e^{-t}$

5) $\frac{100}{\pi} e^{-t}$.

IV) (25 points) Consider Laplace's DE: $\Delta u = 0$ in a square of side 8. Suppose the boundary conditions to be:

$$u(0, y) = u(8, y) = u(x, 8) = 0 \text{ and } u(x, 0) = 4.$$

Compute the solution of the problem.

V) (25 points) The function $g(y) = y$ on $[0, \pi]$ can be extended to be an even function on $[-\pi, \pi]$. Then it possesses the Fourier Series

$$y = \frac{\pi}{2} - \frac{4}{\pi}\left(cosy + \frac{cos3y}{3^2} + \frac{cos5y}{5^2} + \cdots\right), \quad 0 \leq y \leq \pi$$

We can integrate this series term by term from 0 to x.

a) Write down the integrated equation you get (there are no arbitrary constants).

b) By selecting an appropriate point for evaluating both sides, compute the sum of the series

$$1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots.$$